Advanced Composite Materials
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tacm20

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Version of record first published: 02 Apr 2012.

Adv. Composite Mater., Vol. 9, No. 1, pp. 47–58 (2000)

Notched strength evaluation of fabric laminates having a circular hole

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Received 16 December 1998; accepted 13 March 1999

Abstract—modifications are suggested in the Whitney–Nuismer (WN) two stress fracture criteria known as the point stress criterion and average stress criterion for evaluating the notched strength of composite laminates having a circular hole.The linear relationship between the asymptotic value of the center cracked tension plate stress intensity factor and the notched strength of the infinite plate is used for the evaluation of the characteristic lengths to evaluate the notched strength of laminates. The analytical results based on the modified WN fracture models are compared well with the existing test results of fabric laminates of E-glass/epoxy.

Keywords: Notched strength; fabric laminates; circular hole; point stress criterion; average stress criterion; characteristic lengths; critical stress intensity factor.

1. INTRODUCTION

With the advent of composite materials and their wide use in structural applications, it has become necessary to drill holes into the laminates to facilitate bolting or rivetting to the main load–bearing structures. Drilling holes significantly reduces the performance of composites due to the stress concentration which is maximum at the edge of the hole. Whitney and Nuismer [1] presented two stress fracture criteria known as the ‘point stress’ criterion (PSC) and the ‘average stress’ criterion (ASC) for predicting the tensile strength of composite laminates having a circular hole. In the PSC, it is assumed that the failure occurs when the stress over some distance (d_0) away from the notch is equal to or greater than the unnotched laminate strength (σ_0). Similarly, in the ASC, it is assumed that the failure occurs when the average stress over some distance (a_0) ahead of the notch equals the unnotched laminate strength (σ_0). An approximate solution in the form of a polynomial was used to find the normal stress distribution adjacent to the hole in an infinite width plate. Both stress fracture criteria are two parameter models based on the unnotched strength (σ_0) and a characteristic dimension (d_0 or a_0). Pipes *et al*. [2] extended the Whitney–Nuismer (WN) fracture model and introduced a three-parameter notched strength model in which the characteristic length,

$$d_0 = C_\mathrm{p}(R/R_0)^{m_\mathrm{p}}, \tag{1}$$

is assumed as a function of the hole size (R), reference radius (R_0), notch sensitivity factor ($C_\mathrm{p} > 0$), and exponential parameter ($0 < m_\mathrm{p} < 1$).

Kim *et al*. [3] expressed the possibility of varying the notch sensitivity factor (C_p) with the selection of the reference radius (R_0), and considered the characteristic length,

$$d_0 = C_\mathrm{k}(2R/W)^{m_\mathrm{k}}, \tag{2}$$

in the PSC as a function of hole diameter ($2R$), specimen width (W), notch sensitivity factor (C_k), and exponential parameter (m_k). For any material system, the constants in equations (1) and (2) for d_0 are to be determined experimentally from testing unnotched and notched specimens. A minimum of two notched specimen tests is required; normally more tests are performed to take into account the scatter in test results. Strictly speaking, the notched tensile strength of an infinite plate (σ_N^∞) is obtained by multiplying the experimental notched strength of a finite width plate (σ_N) by a correction factor ($K_\mathrm{T}/K_\mathrm{T}^\infty$) and the characteristic length (d_0) is determined from the PSC using the normal stress distribution adjacent to the hole in an infinite plate. Here, K_T and K_T^∞ are the stress concentration factors for finite and infinite width orthotropic plates, which are given by [4]

$$K_\mathrm{T}^\infty/K_\mathrm{T} = 3(1 - D/W)/\left\{2 + (1 - D/W)^3\right\} + 1/2\{(D/W)M\}^6(K_\mathrm{T}^\infty - 3)\left[1 - \{(D/W)M\}^2\right], \tag{3}$$

where

$$\{(D/W)M\}^2 = 1/2\{\sqrt{1 - 8\left[\left\{3(1 - \mathrm{D}/W)/\left\{2 + (1 - D/W)^3\right\}\right\} - 1\right]} - 1\}, \tag{4}$$

$$K_\mathrm{T}^\infty = 1 + \sqrt{2\left(\sqrt{E_{xx}/E_{yy}} - \nu_{xy}\right) + (E_{xx}/G_{xy})}, \tag{5}$$

D ($= 2R$) is the diameter of holes, W is the width of the specimen, E_{xx}, E_{yy} and G_{xy} are the axial, transverse, and shear moduli, respectively, and ν_{xy} is the major Poisson's ratio for the laminate. For an infinite plate, only the hole size is finite and expressing d_0 in terms of R is valid. The reference radius (R_0) in equation (1) was used only to satisfy the requirements of dimensional consistency.

In the present study, a linear relationship between the asymptotic value of the center cracked tension plate stress intensity factor (K_Q) and the notched strength (σ_N^∞) of the infinite plate is used for evaluation of the characteristic lengths (namely,

d_0 for PSC and a_0 for ASC), to evaluate the notched strength of laminates. The analytical results based on the present PSC and ASC are found to be resonably in good agreement with the test results [5].

2. WHITNEY–NUISMER FRACTURE MODEL

The Whitney–Nuismer (WN) model postulates final fracture of a notched composite laminate if the stress in a certain zone exceeds the ultimate strength of the unnotched material.Two different criteria have been formulated.

The point stress criterion (PSC) assumes that failure occurs when the stress, σ_y, over some distance, d_0, away from the discontinuity is equal to or greater than the strength of the unnotched laminate (σ_0):

$$\sigma_y(x,0) = \sigma_0 \quad \text{at } x = R + d_0. \tag{6}$$

The average stress criterion (ASC) assumes that failure occurs when the average stress, σ_y, over some distance, a_0, equals the unnotched laminate strength (σ_0):

$$\sigma_0 = (1/a_0)\int_R^{R+a_0} \sigma_y(x,0)\,\mathrm{d}x. \tag{7}$$

2.1. Circular holes

For an infinite orthotropic plate subjected to a uniform stress, σ_y^∞, applied parallel to the y-axis at infinity, the normal stress, σ_y along the x-axis ahead of the hole can be expressed as [6, 7]

$$\sigma_y(x,0) = (\sigma_y^\infty/2)\Big\{\ 2 + (R/x)^2 + 3(R/x)^4 - (K_\mathrm{T}^\infty - 3)\big[5(R/x)^6 - 7(R/x)^8\big]\Big\}, \quad x > R. \tag{8}$$

Using equation (8) in conjunction with the PSC:

$$\sigma_\mathrm{N}^\infty/\sigma_0 = 2/\big\{2 + \Phi_1^2 + 3\Phi_1^4 - (K_\mathrm{T}^\infty - 3)(5\Phi_1^6 - 7\Phi_1^8)\big\}, \tag{9}$$

$$\text{with } \Phi_1 = R/(R + d_0). \tag{10}$$

Applying the ASC to equation (8):

$$\sigma_\mathrm{N}^\infty/\sigma_0 = 2\big\{(1 + \Phi_2)\big[2 + \Phi_2^2 + (K_\mathrm{T}^\infty - 3)\Phi_2^6\big]\big\}^{-1}, \tag{11}$$

$$\text{with } \Phi_2 = R/(R + a_0). \tag{12}$$

2.2. Straight cracks

The stress in the vicinity of a crack in an orthotropic laminate of infinite width under uniaxial loading, σ_y^∞, is yielding a singularity at the crack-tip. The exact anisotropic elasticity solution for the normal stress ahead of a crack of length $2c$ in an infinite anisotropic center cracked plate under uniform tension, σ_y^∞, is given by [6]

$$\sigma_y(x,0) = \sigma_y^\infty x / \left(\sqrt{x^2 - c^2}\right), \quad x > c. \tag{13}$$

It should be noted that equation (13) is independent of material properties. Because of the singularity of the crack-tip, the concept of a stress concentration factor is replaced by a stress intensity factor. For uniaxial tension (mode I), this factor is defined as:

$$K_{\mathrm{I}} = \sigma_y^\infty \sqrt{\pi c}. \tag{14}$$

The stress distribution, equation (13), in terms of the stress intensity factor is given by

$$\sigma_y(x,0) = K_{\mathrm{I}} x / \sqrt{\pi c (x^2 - c^2)}, \quad x > c. \tag{15}$$

Applying the PSC in conjunction with equation (13) or (15), the notch sensitivity of an infinite laminated plate with a center crack becomes:

$$\sigma_{\mathrm{N}}^\infty / \sigma_0 = \sqrt{1 - \Phi_3^2}, \tag{16}$$

$$\text{with } \Phi_3 = \{c/(c + d_0)\}, \tag{17}$$

From equations (14) and (16), the critical stress intensity factor K_{Q} becomes

$$K_{\mathrm{Q}} = \sigma_0 \sqrt{\pi c (1 - \Phi_3^2)}. \tag{18}$$

Applying the ASC in conjunction with equation (13) or (15), the notch sensitivity ratio results as:

$$\sigma_{\mathrm{N}}^\infty / \sigma_0 = \sqrt{(1 - \Phi_4)/(1 + \Phi_4)}, \tag{19}$$

$$\text{with } \Phi_4 = \{c/(c + a_0)\}. \tag{20}$$

From equations (14) and (19), the critical stress intensity factor K_{Q} becomes

$$K_{\mathrm{Q}} = \sigma_0 \sqrt{\pi c (1 - \Phi_4)/(1 + \Phi_4)}. \tag{21}$$

In both equations (18) and (21), the expected limit of $K_{\mathrm{Q}} = 0$ for vanishingly small crack lengths is reached, while for large crack lengths, K_{Q} asymptotically approaches a constant value ($K_{\mathrm{Q}\infty}$). For the point and average stress criteria, these asymptotic values are respectively

$$K_{\mathrm{Q}\infty} = \sigma_0 \sqrt{2\pi d_0}, \tag{22}$$

$$K_{\mathrm{Q}\infty} = \sigma_0 \sqrt{\pi a_0 / 2}. \tag{23}$$

The Whitney–Nuismer fracture models discussed above for laminated composites with a circular hole and a crack, are two different cases. These fracture models are similar to the inherent flaw model of Waddoups *et al.* [8] in which their model assumes that near the hole there are regions of intense energy of length transverse to the loading direction and the intense energy regions are considered as cracks. Hence, a characteristic size (d_0 or a_0) in each case is considered to be a damage zone immediately ahead of the hole or crack. If the performance of a laminate is affected by the stress concentration, it is defined as 'notch sensitive'. If it is affected by the change of opening shape with a same opening length, the laminate is defined as 'notch shape sensitive'. The fracture data existing on sevaral composite laminates indicates that most laminates are 'notched shape sensitive. The damage zone immdiately ahead of the hole is expected to be different from that of a crack.

It can be observed from equations (9), (11), (18) and (21) that the strength of a laminate decreases with the hole size and the critical stress intensity factor increases with the crack size. The Whitney–Nuismer fracture models for laminated composites with a circular hole or with a crack, are based on the unnotched tensile strength (σ_0) and a characteristic dimension (d_0 or a_0). In each case, the notched tensile strength reduces with the size of discontinuity (hole or crack). Since the behaviour of fracture strength is assumed to be similar for laminated composites with a circular hole or with a crack, it is more appropriate to define the fracture toughness ($K_{Q\infty}$) in terms of σ_0 and the characteristic dimension (d_0 or a_0) as given by equations (22) and (23).

When the fracture stress (σ_N^∞) is less than the unnotched strength of the laminate (σ_0), the relation between the stress intensity factor, $K_{Q\infty}$, and the stress (σ_N^∞) at failure can be written in the form [9]

$$K_{Q\infty} = K_F\{1 - m(\sigma_N^\infty/\sigma_0)\}. \tag{24}$$

The parameters K_F and m in equation (24) are determined by a least-square curve fit to the data of $K_{Q\infty}$, σ_N^∞ and σ_0. The nondimensional value of m in general is greater than zero and less than unity. Whenever m is found to be greater than unity, the parameter m has to be truncated to 1.0 by suitably modifying the parameter K_F with the fracture data. If m is found to be less than zero, the parameter m has to be truncated to zero and the average of $K_{Q\infty}$ values from the fracture data yields the parameter K_F.

When the point stress and the average stress criterion are used, one get two sets of equations. Since the characteristic dimensions, d_0 or a_0, for the two fracture models are different, the values of K_F and $m$in equation (24) become different. Hence, the parameters K_F and m in equation (24) are to be determined separately for the laminated composites having circular holes and cracks. The characteristic length in equation (24) represents a quadratic polynomial in terms of the notched strength of a laminate. The variation of the characteristic length depends on the value of the parameter m in equation (24). When m is equal to zero, the characteristic length is independent of the notch size.

Evaluation of characteristic length from a simple relation (24) for notched tensile strength of composite laminates through Whitney–Nuismer fracture models, has several advantages compared to the exisiting empirical relations. In Whitney–Nuismer fracture models, it is assumed that the characteristic dimension has the same values for all notch sizes, which may not be valid for all materials. Pipes *et al*. [2] introduced equation (1) in which the characteristic length is assumed to be a function of hole size. But the test results presented by Kim *et al*. [3] show variation in the characteristic length with the hole size as well as the width of the plate.

Based on the test results, Kim *et al*. [3] suggested that d_0, given by equation (2), is a function of hole size and width of the plate. The constants in equation (2) presented by them are found to vary with the width of the plate and hence introducing the width (w) in equation (2) in place of the reference radious (R_0) in equation (1) has no additional benefits. When the parameters are changing for each width configuration, then calculating the constants either in equation (1) or in equation (2) may not be useful for the designer to design a component in advance because he has to conduct tests for the intended diameter and width to get the actual notched tensile strength value. Instead, he can perform the test directly for the intended diameter and width to get the actual notched tensile strength value. In such a case analytical modelling may not be useful to designers.

Kim *et al.* [3] also tried to correlate the notched strength with the characteristic length using a linear relationship. The constants in the relation are also found to vary with the width of the plate. Such a relation also will not be directly useful. The fracture data of Ref. [5] is considered here to determine the material constants K_F and m in the linear relationship (24). Using this equation with the determined material constants, the notched tensile strength values have been calculated and are found to be in reasonably good agreement with the test results. The simple relation (24) in the present study will be more useful to the designer for getting more information on notched tensile strength of any diameter to width ratio, through the Whitney–Nuismer fracture models.

In order to distinguish the fracture toughness parameters for both the cases, equation (24) is written separately as

$$K_{\mathrm{QP}\infty} = K_{\mathrm{FP}}\{1 - m_{\mathrm{P}}(\sigma_{\mathrm{N}}^{\infty}/\sigma_0)\} \quad \text{for PSC,} \tag{25}$$

$$K_{\mathrm{QA}\infty} = K_{\mathrm{FA}}\{1 - m_{\mathrm{A}}(\sigma_{\mathrm{N}}^{\infty}/\sigma_0)\} \quad \text{for ASC.} \tag{26}$$

3. RESULTS AND DISCUSSION

Applicability of the linear relationship given by equation (24) between the critical stress intensity factor, $K_{\mathrm{Q}\infty}$, and the failure stress ($\sigma_{\mathrm{N}}^{\infty}$), is examined by considering the notched strength data on E-glass/epoxy fabric laminates of eight different lay-ups [5]. The mechanical properties of the E-glass/epoxy laminates are given in Table 1. The notched tensile strength of an infinite plate ($\sigma_{\mathrm{N}}^{\infty}$) is obtained by multiplying the experimental notched strength of a finite width plate (σ_{N}) by a correction

Table 1.
Mechanical properties of E-glass/epoxy unbalanced plain weave fabric laminates [Resin system: LY 556/HY951. Fibre volume fraction (V_f) = 45%, Average thickness (t) = 1.32 mm]

Notation	Lay-ups	E_{xx} (GPa)	E_{yy} (GPa)	G_{xy} (GPa)	ν_{xy}	σ_0 (MPa)
SS1	$(0)_{4S}$	18.37	16.70	6.82	0.14	370
SS2	$(45, 0)_{2S}$	15.12	11.89	9.10	0.33	280
SS3	$(45, 0_2, 45)_S$	14.37	12.90	7.23	0.33	274
SS4	$(45_2, 0_2)_S$	15.92	10.71	5.95	0.36	263
SS5	$(0, 45)_{2S}$	12.63	11.45	7.76	0.33	257
SS6	$(0_2, 45_2)_S$	13.68	10.96	7.20	0.31	249
SS7	$(0, 45_2, 0)_S$	15.88	12.90	6.52	0.33	244
SS8	$(45)_{4S}$	7.88	6.66	7.20	0.71	150

Table 2.
Comparison of notched strength values of E-glass/epoxy fabric laminate having a circular hole (SS1). Width of the plate = 50 mm; PSC: K_{FP} = 51.64 MPa$\sqrt{m}$, m_P = 1; ASC: K_{FA} = 39.14 MPa$\sqrt{m}$, $m_A = 1$

$D(=2R)$ (mm)	σ_N (MPa) Test [5]	σ_N^∞ (MPa)	Analysis					
			PSC			ASC		
			$K_{QP\infty}$ (MPa$\sqrt{m}$)	σ_N (MPa)	Relative error (%)	$K_{QA\infty}$ (MPa$\sqrt{m}$)	σ_N (MPa)	Relative error (%)
5	184	185.98	22.06	195.14	−6.06	17.28	191.74	−4.21
10	165	172.69	27.62	164.82	0.11	21.20	164.12	0.53
15	142	158.38	28.70	143.46	−1.03	21.55	143.78	−1.25
18	133	156.58	30.68	131.67	1.00	22.97	132.26	0.55
20	128	157.37	32.69	123.88	3.22	24.51	124.58	2.67

factor (K_T/K_T^∞) as defined in equation (3). Substituting the values of σ_N^∞ and σ_0 in equations (9) and (11) and solving it using Newton–Raphson iterative scheme, the characteristic lengths, d_0 and a_0 corresponding to the dimension of the circular hole is determined. The asymptotic values of $K_{Q\infty}$ (namely, $K_{QP\infty}$ and $K_{QA\infty}$) are obtained by using the values of d_0 and a_0 in equations (22) and (23). The asymptotic values of $K_{QP\infty}$ and $K_{QA\infty}$ presented in Tables 2–9 are found to be different for each size of the hole, which indicates that the characteristic dimensions (d_0 or a_0) are not uniform. The parameters K_{FP}, m_P, K_{FA}, and m_A in equations (25) and (26) are determined through a least-square curve fit from the generated data. The linear relations in equations (25) and (26) assumed for PSC and ASC, represent the characteristic dimensions (d_0 and a_0) as quadratic polynomials in terms of σ_N^∞/σ_0.

Table 3.
Comparison of notched strength values of E-glass/epoxy fabric laminate having a circular hole (SS2). Width of the plate = 50 mm; PSC: $K_{FP} = 82.36$ MPa$\sqrt{}$m, $m_P = 1$; ASC: $K_{FA} = 55.96$ MPa$\sqrt{}$m, $m_A = 0.88$

$D(=2R)$ (mm)	σ_N (MPa) Test [5]	σ_N^{∞} (MPa)	Analysis					
			PSC			ASC		
			$K_{QP\infty}$ (MPa$\sqrt{}$m)	σ_N (MPa)	Relative error (%)	$K_{QA\infty}$ (MPa$\sqrt{}$m)	σ_N (MPa)	Relative error (%)
5	185	186.99	25.67	188.44	−1.86	21.88	188.27	−1.77
10	166	173.76	32.81	163.30	1.63	26.94	163.10	1.75
15	144	160.74	36.04	142.60	0.97	28.61	142.59	0.98
18	132	155.72	37.70	130.52	1.12	29.56	130.61	1.06
20	120	148.04	36.79	122.40	−2.00	28.35	122.54	−2.12

Table 4.
Comparison of notched strength values of E-glass/epoxy fabric laminate having a circular hole (SS3). Width of the plate = 50 mm; PSC: $K_{FP} = 77.50$ MPa$\sqrt{}$m, $m_P = 1$; ASC: $K_{FA} = 61.83$ MPa$\sqrt{}$m, $m_A = 1$

$D(=2R)$ (mm)	σ_N (MPa) Test [5]	σ_N^{∞} (MPa)	Analysis					
			PSC			ASC		
			$K_{QP\infty}$ (MPa$\sqrt{}$m)	σ_N (MPa)	Relative error (%)	$K_{QA\infty}$ (MPa$\sqrt{}$m)	σ_N (MPa)	Relative error (%)
5	167	168.79	22.39	182.53	−9.30	18.40	179.11	−7.25
10	163	170.61	32.14	157.90	3.13	26.54	156.95	3.71
15	143	159.61	35.87	137.79	3.65	28.77	138.18	3.37
18	125	147.41	34.97	126.10	−0.88	27.24	126.96	−1.57
20	120	147.96	37.07	118.26	1.45	28.92	119.35	0.55

For any specified hole size, the notched tensile strength, σ_N^{∞}, can be determined from equations (9) and (10), or (11) and (12), which are functions of the characteristic dimensions d_0 or a_0. Using equations (22) and (23) in equations (25) and (26) these characteristic dimensions are expressed as quadratic polynomials in terms of $\sigma_N^{\infty}/\sigma_0$:

$$d_0 = \{1/(2\pi)\}(K_{FP}/\sigma_0)^2\{1 - m_P(\sigma_N^{\infty}/\sigma_0)\}^2, \quad (27)$$

$$a_0 = \{2/(\pi)\}(K_{FA}/\sigma_0)^2\{1 - m_A(\sigma_N^{\infty}/\sigma_0)\}^2. \quad (28)$$

Table 5.
Comparison of notched strength values of E-glass/Epoxy fabric laminate having a circular hole (SS4). Width of the plate = 50 mm; PSC: $K_{\mathrm{FP}} = 78.93$ MPa$\sqrt{}$m, $m_{\mathrm{P}} = 1$; ASC: $K_{\mathrm{FA}} = 65.04$ MPa$\sqrt{}$m, $m_{\mathrm{A}} = 1$

$D(=2R)$ (mm)	σ_{N} (MPa) Test [5]	$\sigma_{\mathrm{N}}^{\infty}$ (MPa)	Analysis					
			PSC			ASC		
			$K_{\mathrm{QP}\infty}$ (MPa$\sqrt{}$m)	σ_{N} (MPa)	Relative error (%)	$K_{\mathrm{QA}\infty}$ (MPa$\sqrt{}$m)	σ_{N} (MPa)	Relative error (%)
5	174	175.87	23.64	179.03	−2.89	20.63	175.91	−1.10
10	148	154.90	28.01	155.64	−5.17	23.08	154.71	−4.53
15	136	151.70	33.31	136.13	−0.10	27.24	136.45	−0.33
18	130	153.07	36.96	124.73	4.05	30.32	125.53	3.44
20	120	147.58	37.00	117.09	2.42	29.95	118.12	1.57

Table 6.
Comparison of notched strength values of E-glass/epoxy fabric laminate having a circular hole (SS5). Width of the plate = 50 mm; PSC: $K_{\mathrm{FP}} = 108.03$ MPa$\sqrt{}$m, $m_{\mathrm{P}} = 1$; ASC: $K_{\mathrm{FA}} = 75.60$ MPa$\sqrt{}$m, $m_{\mathrm{A}} = 0.896$

$D(=2R)$ (mm)	σ_{N} (MPa) Test [5]	$\sigma_{\mathrm{N}}^{\infty}$ (MPa)	Analysis					
			PSC			ASC		
			$K_{\mathrm{QP}\infty}$ (MPa$\sqrt{}$m)	σ_{N} (MPa)	Relative error (%)	$K_{\mathrm{QA}\infty}$ (MPa$\sqrt{}$m)	σ_{N} (MPa)	Relative error (%)
5	187	189.01	27.13	189.25	−1.21	24.55	189.17	−1.16
10	170	177.95	35.17	167.34	1.57	30.52	167.13	1.69
15	148	165.23	38.88	147.63	0.25	32.34	147.59	0.28
18	134	158.13	40.11	135.60	−1.19	32.64	135.67	−1.25
20	128	158.00	42.23	127.37	0.49	34.35	127.51	0.38

Using equation (27) in (9) and eliminating d_0, a nonlinear equation in terms of $\sigma_{\mathrm{N}}^{\infty}/\sigma_0$ and R is obtained for PSC. Similarly, using equation (28) in (11) and eliminating a_0, another nonlinear equation in terms of $\sigma_{\mathrm{N}}^{\infty}/\sigma_0$ and R is obtained. For any specified value of R, $\sigma_{\mathrm{N}}^{\infty}/\sigma_0$ can be determined by solving these nonlinear equations separately through Newton−Raphson iterative scheme.The notched strength of a finite width plate (σ_{N}) is then obtained by multiplying the determined $\sigma_{\mathrm{N}}^{\infty}$ with $K_{\mathrm{T}}^{\infty}/K_{\mathrm{T}}$ for both the cases. Tables 2 to 9 and Fig. 1 give comparison of analytical and test results of the eight different lay-ups. The relative error (%) is defined by $100 \times [1 - \{\sigma_{\mathrm{N}}(\text{analysis})/\sigma_{\mathrm{N}}(\text{test})\}]$. The results are found

Table 7.
Comparison of notched strength values of E-glass/epoxy fabric laminate having a circular hole (SS6). Width of the plate = 50 mm; PSC: $K_{FP} = 82.16$ MPa$\sqrt{}$m, $m_P = 1$; ASC: $K_{FA} = 67.06$ MPa$\sqrt{}$m, $m_A = 1$

$D(=2R)$ (mm)	σ_N (MPa) Test [5]	σ_N^∞ (MPa)	Analysis					
			PSC			ASC		
			$K_{QP\infty}$ (MPa$\sqrt{}$m)	σ_N (MPa)	Relative error (%)	$K_{QA\infty}$ (MPa$\sqrt{}$m)	σ_N (MPa)	Relative error (%)
5	170	171.83	23.78	173.15	−1.85	20.77	170.03	−0.02
10	143	149.68	27.77	151.11	−5.67	22.61	150.16	−5.00
15	133	148.44	33.62	132.40	0.45	27.27	132.71	0.21
18	125	147.39	36.47	121.32	2.94	29.50	122.12	2.30
20	117	144.23	37.28	113.85	2.69	29.90	114.88	1.81

Table 8.
Comparison of notched strength values of E-glass/epoxy fabric laminate having a circular hole (SS7). Width of the plate = 50 mm; PSC: $K_{FP} = 84.14$ MPa$\sqrt{}$m, $m_P = 1$; ASC: $K_{FA} = 70$ MPa$\sqrt{}$m, $m_A = 1$

$D(=2R)$ (mm)	σ_N (MPa) Test [5]	σ_N^∞ (MPa)	Analysis					
			PSC			ASC		
			$K_{QP\infty}$ (MPa$\sqrt{}$m)	σ_N (MPa)	Relative error (%)	$K_{QA\infty}$ (MPa$\sqrt{}$m)	σ_N (MPa)	Relative error (%)
5	167	168.79	23.19	171.98	−2.98	20.46	169.06	−1.23
10	142	148.63	27.45	150.61	−6.06	22.73	149.71	−5.43
15	137	152.85	34.95	132.22	3.49	29.30	132.54	3.26
18	123	144.92	35.55	121.29	1.39	29.13	122.08	0.75
20	117	144.04	37.15	113.91	2.64	30.37	114.92	1.77

to be in reasonably good agreement with each other. The analytical results obtained from PSC are close to those obtained from ASC. The simple linear relationship (24) followed in the present study for determination of characteristic dimension, is useful for notched tensile strength predictions of laminates through Whitney–Nuismer fracture models.

Table 9.
Comparison of notched strength values of E-glass/epoxy fabric laminate having a circular hole (SS8). Width of the plate = 50 mm; PSC: $K_{\mathrm{FP}} = 68.23$ MPa$\sqrt{}$m, $m_{\mathrm{P}} = 0.967$; ASC: $K_{\mathrm{FA}} = 44.94$ MPa$\sqrt{}$m, $m_{\mathrm{A}} = 0.834$

$D(=2R)$ (mm)	σ_{N} (MPa) Test [5]	$\sigma_{\mathrm{N}}^{\infty}$ (MPa)	Analysis					
			PSC			ASC		
			$K_{\mathrm{QP}\infty}$ (MPa$\sqrt{}$m)	σ_{N} (MPa)	Relative error (%)	$K_{\mathrm{QA}\infty}$ (MPa$\sqrt{}$m)	σ_{N} (MPa)	Relative error (%)
5	115	116.24	17.48	114.41	0.51	16.23	114.47	0.46
10	96	100.50	20.13	101.20	−5.42	16.66	101.09	−5.31
15	88	98.34	23.96	89.17	−1.33	19.56	89.15	1.30
18	84	99.38	26.61	81.75	2.67	21.87	81.80	2.62
20	79	97.93	27.51	76.64	2.99	22.40	76.73	2.88

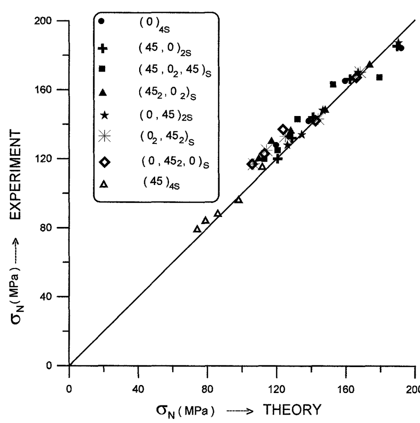

Figure 1. Comparision of analytical (based on PSC) and experimental notched tensile strength, σ_{N} (MPa) values of fabric laminates of E-glass/epoxy.

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